# Game Theory

Introduction to Game Theory

# What is a Game?

- You might not know it, but you play games everyday.
- Game are played in both everyday life and in extremely important social interactions. For example, games theory can be used to describe voting, war, and dating.
- A game of strategy is a game that goes beyond that of just chance. Strategy involves players making decisions to maximize the payoff. For example, a tennis player's strategic skill comes in knowing where to hit the serve. Her physical skill is being able to hit the ball.

# What is a Game?

- Game theory can help you become more successful at playing games; however, as games become more complex with larger strategy sets and time limits, game theory must be coupled with one's own experience. In this sense, game theory is both an art and a science.
- For example, playing chess can be easy if you use game theory for each movement. Looking ahead a few moves adds many more possibilities to think about. Looking ahead many moves becomes almost impossible. The winner is usually the one who can consistently look ahead the farthest.

# What is a Game?

Games have

- 2 or more players
- one or more outcomes (i.e. someone wins, someone loses)
- Outcomes that depend on players strategies.

# What isn't a Game?

- Pure chance games (lotteries, slot machines, etc.)
- Games without strategic interactions between players (Solitaire)

# Some Examples and Stories of Strategic Games

- Sometimes, the best strategy is to randomize. In boxing, throwing a haymaker every punch will not work. Your opponent will catch onto the pattern and eventually be able to react to it. Given that the players increase their odds of winning if their opponent does not know what they will do, each boxer has the incentive to throw different combinations.

# The Princess Bride

- In the movie, The Princess Bride, Westley enters into a competition with Vizzini. Westley is to poison a glass of wine, the Vizzini picks which glass each will drink. If Westley used logic to choose the glass, then Vizzini would be able to work out that logic and force Westley to drink the poison. Therefore, Westley has the incentive to just pick at random.
- Westley ends up poisoning both glasses and killing Vizzini. Vizzini is then playing a game where he has an information disadvantage.

# The GPA Rat Race

- Let's look at a course that is graded on a curve. Only 40% of the students will gets A's. So, you must work harder than your fellow students are working in order to get an A.
- All the students realize this and agree to fail all the exams.
- If one student decides to do slightly better than the rest of the students, he will receive an A. So that student has the incentive to break cooperation. Other students realize this as well and they all try harder.

# Prisoner's Dilemma

- The GPA game is a special class of games known as a "Prisoner's Dilemma."
- The game involves two criminals being questioned by the police. If neither rats the other out, then they both will do no jail time. If one confesses and the other doesn't, then the one who confessed will have a light jail time while the other is locked away for good. If both confess, then both go to jail.
- What would you do if you were one of the criminals?

# Prisoner's Dilemma

- https://www.youtube.com/watch?v=ED9gaAb2BEw

# Prisoner's Dilemma

- The prisoner's dilemma shows that two completely rational agents may not cooperate even though the cooperation is in their best interest.
- The game has been associated with arms races, steroid use in sports, and climate change.

# Why are Professors so Mean?

- I'm mean. I do not allow makeup exams. Or do I? Do I just act mean in order to benefit the students. As students learn that a professor is willingly to give extensions, more and more students will ask for them. Deadlines then start to mean nothing.
- By refusing to take a step on this slippery slope and allowing no extensions, I avoid this problem. By appearing tough, few students will even attempt to ask for an extension, which saves me the pain of having to deny many students.

# Nash

- https://www.youtube.com/watch?v=LJS7Igvk6ZM

# Decision vs Games

- Strategic games-any game that evens strategy, or one in which the outcomes do not depend on just pure chance or pure skill.
- Decisions-an action situation in a passive environment where a person can choose without concern for the reactions or responses of others.
- What is the distinction of these two terms? Can anyone give examples?
- Unless there are two or more players, each of whom responds to what others do, it is not a game

# Decision vs Games

- Economic interactions are often governed by forces of supply and demand.
- Game theory can be used to describe these chaotic markets when the participation of thousands comes down to the interactions of just two players. This may happen due to two reasons:
- mutual commitments
- private information

# Mutual Commitments

- Mutual commitments comes about when two parties are tied together in a market of many people.
- Lets assume that I want to hire a caterer to cater my party. There are thousands of caters, but eventually I hire one. I pay the cater some initial payment while the cater buys some food. We are then tied together.
- The cater might be tempted to cut corners and slack off. I may be tempted to avoid paying after the party has finished.
- This is where strategy enters. The initial contract has to anticipate the other player's incentives.

# Private Information

- Private information concerns one party having information that is not shared or observed by other parties.
- For example, many people want to be hired for a banking job. For a firm, hiring a person has some risk associated with it. The worker might be a bad or good worker. Only the worker knows which type he is and he has the incentive to say he is a good worker regardless of his type.
- The firm treats each hiring as a strategic game and must develop methods to determine the worker's worth.

# Guessing Game

- No talking!
- Choose a number in the interval [0,100]
- The average of all numbers will then be calculated.
- The person whose number choice is closet to 2/3 times the average of all numbers chosen will be declared the winner.

# Guessing Game

- You can eliminate anything above 66 and 2/3 since those guesses cannot possible be 2/3rds the average.
- Once these number are removed from the strategy set for every player, we can limit the choices between [0, 66.6].
- If each player knows this, we can eliminate anything above 44 and 4/9ths for the same reason.
- This process keeps going until we get to the equilibrium of 0.

# Classifying Games

- Simultaneous or Sequential
- Zero sum?
- Repeated game or one play game
- Information?
- Fixed rules?
- Cooperation agreements enforceable?

# Are the Moves in the Game Sequential or Simultaneous?

- sequential-players move one after another, current moves are influenced by the future consequences of those moves.
- simultaneous-players move at the same time, must anticipate your opponents move right now.
- Some games combine both aspects.
- Examples?

# Are the Players' in Total Conflict or Is There Some Commonality?

- zero sum games-one player wins, one player loses. Add the payoffs and they should equal zero.
- constant sum games-generic version of the zero sum games. players are still in direct conflict but the sum no longer has to equal zero. An example of this game is two players splitting up a sum of money.
- Most economic and social games are not zero-sum.
- International trade for example allows both players to be better off.

# Is the Game Player Once or Repeatedly, and with the Same or Changing Opponents?

- Ongoing games build relationships between players. Previous moves then affect later moves.
- Games may be zero sum in the short run but not in the long run.
- Long distance runners use team work for long stretches as they take turns being in front, but that cooperation collapses when you get near the finish line.

# Do the Players Have Full or Equal Information?

- external uncertainty-a player is uncertain about something outside of the game that still might affect the game (ex: weather)
- strategic uncertainty-uncertain about exactly what moves the opponent has made or is moving.
- perfect information-no external or strategic uncertainty
- imperfect information-there is some external and/or strategic uncertainty
- asymmetric information-one player knows more than the other player

# Do the Players Have Full or Equal Information?

- signals-more informed players giving credible proof of their information
- signaling- strategies that use signals
- screening-less informed player taking actions to create situation in which the more informed player will have to reveal information
- screening devices-methods used for screening

# Are the Rules of the Game Fixed or Manipulable?

- Rules can be determined before the game even starts.
- Examples: preventing firms from even entering the market (barriers of entry).
- Often this is referred to as the "pregame."

# Are Agreements to Cooperate Enforceable?

- cooperative games-games in which joint action agreements are enforceable
- noncooperative games-those in which such enforcement is not possible, and individual participation must be allowed to act in their own interests

# Strategies

- Strategies-the choices available to the players
- Strategies include a full list of actions to be taken at every possibility. Strategies are the same as the action taken in a simultaneous game. In sequential games, an action must determined for every possible move.
- ex: If the other does A, then I will do X, but if the other does B, then I will do Y."
- This is a complete plan of action.

# Payoffs

- payoff-the number associated with each possible outcome.
- Higher payoffs indicate better outcomes.
- Expected payoffs-the mathematical or probabilistic or statistical expectation, meaning an average of all possible outcomes, where each is given a weight proportional to its probability.
- If outcome A has payoff 0 and outcome B has payoff 100, then the prospect of a 75% probability of A and a 25% probability of B should have the payoff
$0.75 * 0 + .25 * 100 = 25$

# Rationality

- Rational Behavior-players are perfect calculators and flawless followers of their best strategies.
- Game theory usually assumes rational behavior.
- Maximizing payoffs does not mean players are selfish. Often, their payoffs include the wellbeing of the other players as well, as valued by that player.
- Is assuming players are rational a good assumption?

# Common Knowledge of Rules

- Usually game theory assumes that all players understand the rules of the game and can therefore act rational.
- Rules consist of lists of players, the strategies available to each player, the payoffs of each player for all possible combination of strategies pursued by all the players, and the assumption that each player is a rational maximizer.

# Equilibrium

- Equilibrium-each player is using the strategy that is the best response to the strategies of the other players.
- Equilibrium does not mean that everything is best for all players.

# Dynamics and Evolutionary Games

- Evolutionary- an approach to games that allows for a dynamic process in which strategies that proved to be better in previous plays of the game are more likely to be chosen in later plays.

# Observation and Experiment

We can find out the reality of strategic interactions in two ways:

- by observing them as they occur naturally
- by conducting special experiments that help us pin down the effects of particular conditions

# The Uses of Game Theory

- explanation
- prediction
- advice or presciption